Micro C exam, June 2012, Solutions

Thomas Jensen

1. (a) Let p denote the probability that player 1 plays A and let q denote the probability that player 2 plays A. In a mixed NE each player must be indifferent between his pure strategies, so we get the following equations:

$$\begin{aligned} Cq &= 1-q \\ -p &= -C(1-p) \end{aligned}$$

From these equations we get

$$\begin{aligned} p &= \frac{C}{1+C} \\ q &= \frac{1}{1+C} \end{aligned}$$

The probability of the outcome (A, A) is

$$pq = \frac{C}{(1+C)^2}$$

The expected payoff for player 1 is

$$pq \cdot C + (1-p)(1-q) \cdot 1 = \frac{C^2}{(1+C)^2} + \frac{C}{(1+C)^2} = \frac{C}{1+C}$$

(b) The pure strategy NE can be found by underlining the best responses in the tri-matrix. There are three pure strategy NE:

$$(A_1, A_2, A_3),\ (A_1, B_2, B_3),\ \text{and}\ (B_1, A_2, B_3)$$

2. (a) See the game tree on the final page. There is one subgame, it starts at the decision node of player 2 and includes the rest of the game.

(b) There are two pure strategy subgame perfect Nash equilibria:

$$(P, D, L) \text{ and } (E, U, R)$$

(c) There is one pure strategy Nash equilibrium that is not subgame perfect:

$$(E, D, R)$$

3. (a) Maximization problem for country i ($i, j \in \{Y, Z\}$, $j \neq i$):

$$\max_{s_i \in [0,1]} \; s_i + s_j - s_i s_j - s_i^2$$

Use the first order condition to find the best response function:

$$s_i = \frac{1}{2} - \frac{1}{2} s_j$$

Thus the conditions for (s_Y^*, s_Z^*) to be a NE are:

$$\begin{aligned} s_Y^* &= \frac{1}{2} - \frac{1}{2} s_Z^* \\ s_Z^* &= \frac{1}{2} - \frac{1}{2} s_Y^* \end{aligned}$$

Solve this system of equations to get:

$$s_Y^* = s_Z^* = \frac{1}{3}$$

(b) The social optimum when the two countries contribute equally can be found by solving the following problem:

$$\max_{s \in [0,1]} \; 2(s + s - ss) - 2s^2$$

By the first order condition we get:

$$\bar{s} = \frac{1}{2}$$

Thus we see that $\bar{s} > s_i^*$. So in the NE the countries use too little of their military capacity relative to the social optimum. This is because of positive externalities. When decisions are made independently country i does not take into account the positive effect that an increase in s_i has on the utility of country j.

(c) Trigger strategy for country i:

- Play $s_i = \frac{1}{2}$ if $t = 1$ or if the outcome of all previous stages was $s_Y = s_Z = \frac{1}{2}$
- Play $s_i = \frac{1}{3}$ otherwise

Best response for country i to $s_j = \frac{1}{2}$ (use the best response fct from question a):

$$s_i = \frac{1}{2} - \frac{1}{2}(\frac{1}{2}) = \frac{1}{4}$$

Payoff for country i in the NE, the social optimum, and when best responding to $s_j = \frac{1}{2}$:

$$\begin{aligned}
\pi^{NE} &= \frac{1}{3} + \frac{1}{3} - (\frac{1}{3})^2 - (\frac{1}{3})^2 = \frac{4}{9} \\
\pi^{SOC} &= \frac{1}{2} + \frac{1}{2} - (\frac{1}{2})^2 - (\frac{1}{2})^2 = \frac{1}{2} \\
\pi^{DEV} &= \frac{1}{4} + \frac{1}{2} - \frac{1}{4}\frac{1}{2} - (\frac{1}{4})^2 = \frac{9}{16}
\end{aligned}$$

Thus the inequality for the trigger strategies to constitute a SPNE is:

$$\begin{aligned}
\sum_{t=1}^{\infty} \delta^{t-1}\pi^{SOC} &\geq \pi^{DEV} + \sum_{t=2}^{\infty} \delta^{t-1}\pi^{NE} \\
&\Longleftrightarrow \\
\sum_{t=1}^{\infty} \delta^{t-1}\frac{1}{2} &\geq \frac{9}{16} + \sum_{t=2}^{\infty} \delta^{t-1}\frac{4}{9} \\
&\Longleftrightarrow \\
\frac{1}{2(1-\delta)} &\geq \frac{9}{16} + \frac{4\delta}{9(1-\delta)}
\end{aligned}$$

(This inequality reduces to $\delta \geq \frac{9}{17}$).

4. (a) See figure 4.2.8 and/or 4.4.4 in Gibbons. The most important point is that $e(H) = e_S > e^*(H)$, the high type chooses a higher level of e than in the full info benchmark (otherwise the low type would imitate him).The low type behaves as in the full info benchmark: $e(L) = e^*(L)$.

   (b) The only separating PBE is:

$$[(R, L), (d, u), p = 0, q = 1]$$

   I.e., type 1 sends the message R, type 2 sends the message L. After seeing the message L, the receiver chooses the action d. After seeing the message R, he chooses the action u.

2. (a) Game tree